Indefinite Information Processing
in Ever-expanding Universes

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I. INTRODUCTION

There have been a number of investigations into the possible future cosmological constraints on information processing in the universe [1–6]. Information processing is regarded as a necessary condition for life by all commentators and as a sufficient condition by some. Thus, if information processing were to become impossible in the future, 'life' would die out. This simple verdict hides all manner of subtleties. It requires us to understand what we mean by 'time', what we mean by 'impossible', and what we mean by 'die out'. For example Dyson [1] considers information processing to 'tick' in comoving proper time whereas Barrow and Tipler [2] consider the merits of a curvature or York time tied to the structure of spacetime geometry for the pulse of abstract life. Some studies consider life to die out if only a finite number of bits of information can be processed to the future. However, Dyson, has stressed the effectiveness of hibernation in extending life and the divergent properties of the harmonic series might thereby offer a means for life to continue forever. Yet it is the need to find plausible physical processes that can store and process information that is crucial to these speculations.

Most investigators consider the sources of free energy that are made available by transitions between elementary particle states [1,2,7,8], proton decay [6], gravitating systems of stellar or post-stellar objects [1,4,9], or black hole evaporations [6,4]. However, as Barrow and Tipler

stressed, the most abundant form of free energy in the late stages of a generic ever-expanding universe is in the form of gravitational waves. In effect, intelligent 'life' can extract tidal energy from the expansion of the universe.

Simple processes for extracting energy from stellar systems in an isotropically expanding universe are not effective for the indefinite survival of information processing when the cosmological constant is zero [1,4]. Worse still, if the cosmological constant is positive, as observations suggest, then information inevitably dies out [2]. Here, we want to show that if small anisotropies in the expansion of the universe are taken into account then it is possible for information processing to continue into the infinite future and to process an infinite number of bits of information. The source of free energy derives from small anisotropies in the three-curvature of the universe. They create a distinctive form of anisotropic expansion that is stable at late time. If a sphere of massless particles were set up in the universe then it would steadily deform into an ellipsoid and the temperatures of orthogonally moving photons would become unequal. Temperature gradients would be created and useful work could be extracted for information processing by exploiting these temperature gradients for arbitrarily small anisotropy.

We will first derive a general limit on entropy production in an expanding universe and then display some explicit examples of almost-isotropic open universes that permit an infinite amount of entropy to be produced to the future. We stress that these anisotropic universes are general in the sense that they are stable in the class of all spatially homogeneous cosmological models.

II. IRREVERSIBLE THERMODYNAMICS

In relativistic cosmology, fluids are usually taken to be perfect fluids in thermal equilibrium. These fluids generate no entropy by frictional heating. However, real fluids behave irreversibly and, compared to reversible thermodynamics, irreversible thermodynamics is poorly understood. The second law of thermodynamics requires that for any physical process the total entropy cannot decrease:

$$\Delta S \geq 0.$$

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In general, the expression for the increase of the entropy is not known, but for a reversible process we have the first law of thermodynamics

$$TdS = dU + pdV - \mu dN.$$

This equation is valid for systems in equilibrium but for irreversible processes the equality has to be replaced by an inequality and the quantities are no longer exact differentials.

Some useful relations have been derived for the increase of entropy in cosmological models, especially when they are close to equilibrium. Consider a dissipative term [10]

$$\mathcal{D} = 3H\Pi + q^\mu_{\ ;\mu} + \dot{u}_\mu q^\mu + \sigma^{\mu\nu}\pi_{\mu\nu}$$

where Π is the bulk viscous pressure, H is the Hubble factor, q^μ is the energy flux in the particle frame, u^μ is the velocity four-vector, $\sigma^{\mu\nu}$ is the shear tensor, and $\pi_{\mu\nu}$ is the trace-free anisotropic stress.

The entropy production in a dissipative model can be calculated using the second law of thermodynamics. For a close-to-equilibrium process the first law is

$$Tn\dot{S} = -\mathcal{D}. \tag{1}$$

Hence, for the second law to be valid, we have to assume that $\mathcal{D} \leq 0$. The entropy in a comoving volume of the fluid is given by

$$\mathcal{S} = a^3 nS$$

where a is the geometric-mean expansion scale factor of the cosmological model. Integrating eq. (1) it follows that the growth in the entropy for the comoving volume over a proper time interval $t_0 \longrightarrow t$ is [10]

$$\mathcal{S}(t) = \mathcal{S}_0 - \int_{t_0}^{t} \frac{a^3}{T}\left(3H\Pi + q^\mu_{\ ;\mu} + \dot{u}_\mu q^\mu + \sigma^{\mu\nu}\pi_{\mu\nu}\right)dt.$$

The term $3H\Pi$ is a bulk viscous heating term and is the only contributor to $\mathcal{S}(t)$ if the expansion is isotropic, while $q^\mu_{\ ;\mu} + \dot{u}_\mu q^\mu$ is contributed by thermal conductivity. In this paper, we will only consider the last term, $\sigma^{\mu\nu}\pi_{\mu\nu}$. This is the shear stress term which describes the dissipative effects due to tidal shear and pressure anisotropy. Thus, we will assume $\mathcal{D} = \sigma^{\mu\nu}\pi_{\mu\nu}$, and the entropy production considered will be a lower bound on the total that could be produced by including other transport processes. In order to process an infinite amount of information as $t \to \infty$ it is necessary for entropy production to be able to increase without bound as $t \longrightarrow \infty$. 'Something' would always be happening in such universes and computations could be done which exploit the scope for indefinite entropy production. The disequilibrium process which gives rise to the entropy production provides the physical basis for the information processing. The effectiveness of this machine is strongly limited by the laws of thermodynamics but to give an upper estimate of the entropy generated by such a machine we can apply the second law of thermodynamics. In terms of information theory, the entropy of a statistical ensemble is just the information needed to completely describe the microscopic state of the system. Information processing and entropy generation are therefore closely related. If we process an amount ΔI of information, the entropy increases with $\Delta S = k_B \ln 2 \Delta I$. Thus this estimate is, up to a constant factor, also an estimate of the amount of information one can process.

III. AN UPPER BOUND ON THE PRODUCTION OF ENTROPY

We will first derive a bound on entropy production from dissipative fluids. The bound will be derived from a theorem by Stewart [11,12]. We will assume the following:

- The Ricci three-curvature scalar is non-positive: This is true for all spatially homogeneous cosmological models except for those of Bianchi type IX and Kantowski-Sachs types.

- The speed of sound for the fluid in every spatial direction is smaller or equal to the speed of light.

The first of these assumptions ensures that the universe is ever-expanding; the second is equivalent to saying that the matter obeys the dominant energy condition.

To acquire the desired entropy-generation bound we have to maximize the function $|\sigma^{\mu\nu}\pi_{\mu\nu}|$ under the conditions $\pi^\mu_{\ \mu} = \sigma^\mu_{\ \mu} = 0$ and $\rho_r \geq |p_\mu|$ where the p_μ are the principal pressures of the dissipative fluid. We can always find a frame where $\pi_{\mu\nu}$ is diagonal, and choosing such a frame we see that $p_r + \pi_{\mu\mu} = p_\mu$ (no summation). Hence, the criterion $\rho_r \geq |p_\mu|$ implies

$$-\rho_r - p_r \leq \pi_{\mu\mu} \leq \rho_r - p_r \tag{2}$$

We will also assume that the anisotropic stress is of ultra-relativistic origin[1], thus we will assume that $p_r = \frac{1}{3}\rho_r$ for the dissipative fluid. The anisotropic stress will now obey the bound

$$|\sigma^{\mu\nu}\pi_{\mu\nu}| \leq \frac{4}{9}\Omega\left(1 - \Omega\right)^{\frac{1}{2}}\theta^3 \tag{3}$$

where $\theta = \dot{V}/V$ is the volume expansion factor and Ω is the total expansion-normalized matter-density which obeys $0 \leq \Omega \leq 1$.

There are some special cases worth noting. First, in a flat FRW universe we have $\Omega = 1$. Hence, in this case the

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bound is zero; the universe can support no anisotropic stresses. Second, in a vacuum Kasner universe $\Omega = 0$, which also implies that the bound is zero; there are no dissipative fluids which could process information.

One might wonder if it is possible to have an equality sign in eq. (3). If we were to extract entropy from these dissipative processes, the second law of thermodynamics implies $\sigma^{\mu\nu}\pi_{\mu\nu} \leq 0$ (see e.g. [10]). Then it is possible to show that for a flat universe, say, with only a dissipative fluid present, we have at late times[2] $\Omega(1-\Omega)^{\frac{1}{2}} \to 0$. Hence, for our purposes, the bound (3) will decrease faster than θ^3 at late times for flat universes.

The bound (3) therefore places a serious restriction on the information processing capacity of an expanding universe. Typically, $\theta \propto t^{-1}$ at late times, and hence the bound (3) decreases faster than t^{-3}. An expansion of the universe caused by, for example, inflationary fluids, will smooth out the anisotropies too rapidly (i.e. $\Omega \to 1$) to allow for infinite information processing; while a flat dust-dominated universe expands too slowly to compensate for the t^{-3} decrease of the bound (3). Effectively, information processing therefore requires a universe that expands faster than the Friedmann dust model but slower than the curvature-dominated Milne model.

IV. PLANE-WAVE SPACETIMES

An interesting set of solutions to the Einstein field equations with the required properties for information processing are the plane-wave solutions of type VII_h. They are future attractor solutions for a large class of open universe models which contain the open Friedmann universe as a special case [14,15]. They are exact vacuum solutions of both the Einstein equations and the linearised Einstein equations. In the special case where they are isotropic they reduce to the Milne universe but in general they are anisotropic and possess both expansion shear and anisotropic spatial curvature. They are members of the most general family of spatially homogeneous universes and are attractors for the late-time evolution of Bianchi VII_h universes. In particular, they describe what happens to perturbed open Friedmann universes at late times when the matter content satisfies $\rho + 3p > 0$ [16,17,14]. Thus, they are anisotropic universes which are stable into the future for a large class of different matter configurations. This means that general spatially homogeneous open universes will evolve towards a state where the Weyl tensor will dominate the Ricci tensor at late

times [18]. Hence, most of the "energy" in the gravitational field is stored in the shear and the three-curvature anisotropy of the spacetime. This is exactly the feature we need if we want to be able to generate an infinite amount of entropy by information processing to the future in an ever-expanding universe. The mean expansion scale factor of the type VII_h plane-wave universes is [19]

$$a(t) \propto t^{1/(1+2\Sigma)}, \qquad (4)$$

where $\Sigma \equiv \sigma/H$ is the ratio of the shear to the mean Hubble expansion rate, and for these plane-wave solutions of Einstein's equations, Σ is a constant that satisfies $0 \leq \Sigma < 1$. When $\Sigma = 0$ we recover the isotropic vacuum Milne universe with $a \propto t$. Note the behaviour of the mean scale factor. The universe expands more slowly as Σ increases and it becomes more anisotropic due to the effects of anisotropic three-curvature. The bounding case of $\Sigma = 1$ would correspond to a universe expanding at the same rate as a spatially-flat dust-filled universe even though the three-curvature is negative and the matter content is dynamically insignificant. This corresponds to the limiting case of the maximum shear anisotropy that is permitted in a cosmological model with positive matter density. This situation of maximal anisotropy for an expanding universe corresponds to *constant* σ/H and hence to $\sigma \propto t^{-1}$, [2,20]. Asymptotic increase in σ/H with time is not possible. The generic late-time asymptote of spatially homogeneous universes that include the open Friedmann universe is asymptotic to this behaviour as $t \to \infty$ [14]. The combination of slower expansion rate and constant shear distortion is what create new possibilities for information processing.

Before we look at the consequences of this generic plane-wave asymptote for information processing to the far future, we should note the circumstances in which our conclusions do not apply. The plane-wave asymptotes are not achieved if there exists a positive cosmological constant, Λ. In general, $\Lambda > 0$ leads to the expansion approaching a de Sitter state with small perturbations which are seen to die away exponentially rapidly within the event horizon of a geodesically moving observer. This makes the eventual extinction of information processing inevitable [2]. The expansion and curvature anisotropies that are needed to sustain temperature anisotropies all die away faster than the (constant) Λ stress in accord with the cosmic no hair theorem [21–23]. Similar pessimistic conclusions for indefinite information processing are expected to hold when the expansion is dominated by stable forms of quintessence with $\rho+3p < 0$ at late times. Here, we shall consider the quite different scenario that results if there is neither stable quintessence nor a positive cosmological constant at late times, so a plane-wave asymptote of the form (4) is approached as $t \to \infty$.

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[2] There are other solutions $(1-\Omega)^{\frac{1}{2}} = \text{const}$ with the time. The negative magnetic solution of Jacobi (13). These solutions however do not obey $\sigma^{\mu\nu}\pi_{\mu\nu} \leq 0$ and Λ cannot produce entropy. In fact they expand adiabatically.

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V. INFORMATION PROCESSING IN AN ANISOTROPIC UNIVERSE

Let us consider an irreversible process which exploits the tidal effect of the shear to generate entropy for the benefit of a civilisation. In principle, this process can be used to drive an information-processing machine so long as the shear energy does not decay too rapidly. Consider a uniform sphere of expanding photons sent out at a given time t_0. Due to the shear in the expansion, photons travelling in different directions will suffer different red shifting at times later than t_0. The spherical distribution of photons will steadily be distorted into an ellipsoid. Hence, if photons were sent out with the same temperature, T_0, the temperature of the photons will be a function of both the time and the direction in which they are traveling. The temperature will therefore be

$$T(t, \theta, \phi) = \bar{T}(t)\Omega(t, \theta, \phi)$$

in suitable chosen coordinates. Here, $\bar{T}(t) \equiv \int_{S^2} T(t, \theta, \phi)d\theta d\phi$ is the average temperature of the photons. Hence, since collisionless photons develop an anisotropic momentum distribution, temperature gradients form which can be used to do work. Therefore, in principle, we can construct an information processor, driven by temperature gradients of radiation.

Thermodynamics is most successful and predictive for processes close to equilibrium and when applied to processes where a temperature can be defined. At late times the universe may be very far from equilibrium and it is unlikely that a temperature can be defined in a simple way. Massless and massive particles will eventually become collisionless in an open universe and the decays of unstable particles will create mixtures of non-equilibrium distributions with different mean energies. Any anisotropy in the expansion will act to transform Planckian distributions into non-Planckian distributions. This complicated thermodynamic behaviour makes the study of the asymptotic evolution of spatially flat universes very difficult to determine because their dynamics are sensitive to the distribution of matter and radiation they contain. However, open universes are simpler to deal with. Asymptotically, gravitationally attractive forms of matter will have a negligible effect on the expansion dynamics and the universe will be increasingly well described by a vacuum solution of the gravitational field equations (which we assume to be those of general relativity). In this situation only the role played by the matter and radiation in generating entropy need be considered.

Consider information processing driven by irreversible processes in radiation. If the anisotropic pressure stresses of the radiation fluid are $\pi_{\mu\nu}$, then close to equilibrium, we will maximise entropy generation when [10]

$$d\mathcal{S}_{\max} \propto \frac{a^3(t)}{T(t)}\sigma^{\mu\nu}\pi_{\mu\nu}dt.$$

If we are able to construct a machine with efficiency $\epsilon(t)$, the machine generates an amount of entropy given by

$$d\mathcal{S}_M \propto \frac{a^3(t)}{T(t)}\sigma^{\mu\nu}\pi_{\mu\nu}\epsilon(t)dt.$$

Now, assume that asymptotically $\sigma_{\mu\nu} \propto t^{-\alpha}$, $\pi_{\mu\nu} \propto t^{-\beta}$ and $\epsilon \propto t^{-\gamma}$ at late times. Let us also use the average background temperature $\bar{T}(t)$ as a lower limit on the temperature. Hence, $T(t) \approx \bar{T} \propto a^{-1}$. Thus, at late times, we get

$$d\mathcal{S}_M \propto a^4(t)t^{-\alpha-\beta-\gamma}dt.$$

For the plane-wave solutions $a(t) \propto t^{1/(1+2\Sigma)}$, where $0 < \Sigma \leq 1$ and $\alpha = 1$. If we assume that the machine takes advantage of stresses of electromagnetic origin of the type $\pi_{\mu\nu} = C_{\mu\nu}\rho_r$ [24,25], then $\beta = 4/(1+2\Sigma) - \delta$, where δ is a small constant obeying[3] $2(1-2\Sigma)/(1+2\Sigma) > \delta$, [18]. Thus we get indefinite information-processing so long as the efficiency parameter obeys the weak bound

$$\gamma \leq \delta.$$

The constant δ arises from the $\sigma_{\mu\nu}\pi^{\mu\nu}$ term, and should be positive due to entropy arguments. Thus, as long as the efficiency parameter γ is less or equal to δ, the machine can process an indefinite amount of information in these plane-wave futures. More specifically, we have at late times

$$\mathcal{S}_M \propto \begin{cases} \ln t, & \gamma = \delta, \\ t^{\delta-\gamma}, & \gamma < \delta. \end{cases}$$

Hence, the efficiency of the machine can actually approach zero at late times, but still manage to generate an unbounded amount of information. Note also that the parameter Σ can be arbitrarily small and positive. In the limit where $\Sigma \to 0$, we recover the isotropic Milne universe. Hence, these solutions can be arbitrary close to isotropy and still generate infinite amounts of entropy as $t \to \infty$.

Another way of seeing why it is possible to process information indefinitely in these universes is to look at the evolution of the Weyl curvature as $t \to \infty$. As universes asymptotically approach the plane-wave asymptotes there will be infinite number of oscillations of the Weyl curvature to the future (see [14] and [26] for demonstrations of these late-time oscillations in other spatially homogeneous universes of Bianchi types VII_0 and VIII. We suggest that infinite information processing should also be possible in universes of these types by utilising the Weyl curvature oscillations).

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[3] h o nd r e fro t he req re e t h t he p ne- te o on o d e ft re t e

This analysis has used massless particles as the source of entropy generation. If we were to use the lightest stable massive particles then they would ultimately be non-relativistic and their temperatures would fall, on the average, as $T \propto a^{-2}$. Their momentum distribution would become anisotropic and it would be easier to obtain divergent entropy production from this anisotropy than from that in massless particles as $t \to \infty$, because their averaged enegy density redshifts away more slowly than that of a trace-free gas of collisionless radiation.

We have taken an averaged approach to the evolution of the 'temperature' of the particles. We can look in more detail at the development of anisotropy in the temperature distribution. In a type VII$_h$ plane-wave spacetime photons moving in orthogonal directions will develop a temperature anisotropy pattern corresponding to a twisted quadrupole that is focussed by the negative spatial curvature into a region of the sky determined by the radius of curvature [27–30]. The temperature of photons that move in orthogonal directions will fall off as $T \propto t^{-1}$ in two orthogonal directions and as $T \propto t^{-2/(1+\Sigma)}$ in the third. In a slightly inhomogeneous situation the alignment of these axes will be position dependent, but in all cases an extremely ellipsoidal distribution will result, with accompanying temperature gradients.

This conclusion is only possible if there is no positive cosmological constant or stable stress with $\rho + 3p < 0$. Both would drive the expansion anisotropy to zero too rapidly for information processing to persist with $a(t) = \exp(t\sqrt{\Lambda/3})$ in the cosmological constant case.

Predictions as to the asymptotic behaviour of cosmological models as $t \to \infty$ are of course very precarious. The tiniest of changes, however insignificant now, can dominate the ultimate behaviour. For example, very slow variations in the supposed constants of Nature could ultimately be the determining factor at late times [31].

Other corrections to Einstein's equations may also be important, although we note that the plane-wave spacetimes are stable exact solutions of gravity theories more general than Einstein's that are generated from a Lagrangian that is an analytic function of the scalar curvature so long as the cosmological constant is zero [32].

The models we have been considering have been spatially homogeneous. We do not expect this to be a significant restriction at late times. Inhomogeneities will freeze out and locally the universe will look increasingly like a homogeneous model; any inhomogeneities in the shear anisotropy will only increase the scope for entropy production by enhancing the local temperature gradients.

A more interesting constraint arises from the global topology of the universe. We have been assuming that the topology of the open universes is the natural $\mathbb{R}^3$ topology and so their spatial volume is infinite. The conclusions change if their 3-spaces are compactified. The Mostow rigidity theorem ensures that Bianchi type VII$_h$ universes must be isotropic if they possess a compact spatial topology [33,34]. In that case the Lukash plane waves are not solutions of the Einstein equations unless $\Sigma = 0$, and the possibility of indefinite information processing in type VII$_h$ universes is removed, but it appears to remain in cosmologies of Bianchi type VIII [33].

VI. CONCLUSIONS AND OUTLOOK

We have shown that if there is no positive cosmological constant (or similar stable stress with $\rho + 3p < 0$) and the topology of space is non-compact, then it is possible to generate an infinite amount of entropy in an ever-expanding open universe by taking advantage of cosmological shear and curvature anisotropy in universes with generic asymptotic behaviour. To show this we used the plane-wave solutions which are general in the sense that they are stable late-time attractors in the class of spatially homogeneous universes. An important feature of these spacetimes is that they are Weyl-curvature dominated as $t \to \infty$ [18]. They allow indefinite information processing to continue by extracting the shear energy created by anisotropic expansion and three-curvature. Ultimately, information processing in these universes is made possible by the effects of gravitational waves on the temperature distributions of collisionless particles. Civilisations who are technologically agile enough to make use of these gradients and space-time oscillations will be living proof of the unlimited cosmological potential of tidal power.

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